This snippet is taken from section 4 of my paper, which identifies bounds for the parameters of the FitzHugh-Nagumo system of ODEs (given below) to help speed up the computational time of the parameter-estimation gradient descent algorithm. Not included: above, sections 4.1-4.3 derive a bound on α (used in this proof) and introduce the concept of a fast-slow dynamical system from Perturbation Theory.

$$\frac{dv}{dt} = v(1-v)(v-\alpha) - w + I_{ext} \qquad \frac{dw}{dt} = \varepsilon(v - \gamma w)$$

4.4 Gamma (γ)

Next, γ physiologically represents the damping of the recovery process — i.e., how quickly the slow variable follows the fast variable. The lower bound for γ is 0, as a negative γ would cause w — and subsequently v — to diverge, which would contradict the fact that biological systems are inherently self-regulating and strive to maintain a homeostasis.

To find an upper bound for γ, we first look to the Jacobian:

$$J(v,w) = \begin{pmatrix} -3v^2 + 2(a+1)v - \alpha & -1 \\ \varepsilon & -\varepsilon\gamma \end{pmatrix}$$

For a two-dimensional system, sufficient conditions for the stability of an equilibrium (at that equilibrium) are: 1) the trace of the Jacobian is negative and 2) the determinant of the Jacobian is positive. Solving for the first condition yields a lower bound, but this is redundant as it is only marginally better than the one that has been already established. Instead, in the spirit of the second condition, we consider the determinant:

$$\det(J) = \left[-3v^2 + 2(\alpha+1)v - \alpha\right][-\varepsilon\gamma] - (-1)(\varepsilon)$$

Factoring out ε yields:

$$\det(J) = \varepsilon \left[\gamma \underbrace{(3v^2 - 2(\alpha+1)v + \alpha)}_{K(v)} + 1\right]$$

Because $\varepsilon > 0$ (discussed further in section 4.5), the sign of $\det(J)$ is dependent on $\gamma K(v) + 1$. The derivative of $K(v)$ is:

$$K'(v) = 6v - 2(\alpha+1)$$

With critical point:

$$v = \frac{\alpha+1}{3}$$

Because $K''(v) = 6 > 0$, this critical point is also a minimum of $K(v)$. Evaluating $K(v)$ here yields:

$$K\left(\frac{\alpha+1}{3}\right) = 3\left(\frac{\alpha+1}{3}\right)^2 - 2(\alpha+1)\left(\frac{\alpha+1}{3}\right) + \alpha = -\frac{\alpha^2 - \alpha + 1}{3}$$

Evaluating this at the upper bound of $\alpha = 0.18$ (to minimize $K(v)$) yields $K(\frac{\alpha+1}{3}) \approx -0.284$. And imposing $\det(J) > 0$, we obtain:

$$\gamma \cdot (-0.284) + 1 > 0$$
$$\implies \gamma < \frac{1}{0.284} \approx 3.52$$

Thus, $\boxed{0 \le \gamma \le 3.52}$